

Stirling engines using working fluids with strong real gas effects

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ABSTRACT

Real gas effects typical of the critical region of working fluids are a powerful tool to increase the energy performances of Stirling cycles, mainly at low top temperatures. To carry out the compression near the critical region the working fluids must have a critical temperature near environmental conditions and the use of organic working substances (pure or in suitable mixtures) as a matter of fact begins compulsory. The moderate thermal stability of the organic working fluids limits the maximum temperatures to 300–400 °C and as a consequence, the achievable cycles efficiencies result rather low. Carbon dioxide, with a critical temperature of 31 °C, is, among the traditionally inorganic gases, an exception and is considered here in comparison with organic substances. But the good thermodynamics of the cycles allows, in the considered cases, conversion efficiencies of about 20%, with good specific powers. The good energy performance of real gas Stirling cycles is obtained at the cost of high maximum cycle pressure, in the range of at least 100–300 bar. These high pressures nevertheless have large positive effects on the heat power transferred per unit of pumping mechanical power, and the low top temperatures have a positive influence on the material problems for the hottest engine parts.

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1. Introduction

The Stirling engine works according with a closed and regenerative thermodynamic cycle. In a traditional Stirling engine the volume variations are the consequence of the alternate motion of pistons carrying out the compression and the expansion of the working gas at two different temperatures.

The cyclic variation of the volume produces mass fluxes inside the engine and the net effect is the production of useful work equals to the net heat exchanged, [1].

The engine can use every kind of fuels. For example, natural gas, coal, biomass or solar energy, [2–5]. It is quieter than the traditional internal combustion engines. Owing to the internal regeneration, it may potentially reach very high thermodynamic efficiencies: the ideal cycle has in fact the same efficiency of the Carnot cycle operating between the same temperatures. In the ideal Stirling thermodynamic cycle the temperature increase of the working gas, passing from the cold region to the hot one, is induced by the regenerator. This is one of the main reasons that makes the Stirling particularly suitable for the realization of engines of small power (some kW, for example). On the contrary, in the thermodynamic cycles like the Carnot cycle, without regenerator, the increasing of temperature (and pressure) of the working fluid is totally assigned

to the turbomachineries, that are very difficult to realize if the engine is of very small size and power.

Furthermore, the Stirling engines are in several aspects similar to the thermoacoustic engines, [6], and, on the whole, the basic conclusions of the analyses carried out for a type are valid for the other too.

The primary interest in the development of small power engines is in the distributed generation of heat and power, for example for domestic application. Small, cheap and highly reliable solar or biomass powered engines may even have a large market in the undeveloped countries. Often, an high specific power is a fundamental prerequisite.

In the field of the distributed power generation, as a thermal machine, the Stirling engine is in competition with Rankine cycles and micro-gas turbines. Among the advantages of the Rankine cycles is their good thermodynamic efficiency even at moderate high temperatures, provided that an adequate working fluid is selected; Brayton cycles, even in closed cycles, are also potentially very interesting but in any case the design of efficient small turbomachines results very difficult.

In modern Stirling engines, the working fluids are usually helium or hydrogen for engines of high performances. Helium and hydrogen, by virtue of their good thermo-physical characteristics, allow the design of engines with high heat fluxes and moderate pressure losses, but on the other hand, they impose severe sealing problems. Air was repeatedly proposed for engines of simple design and relatively low specific power with low speed, [7,8]

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Nomenclature*Symbols*

C_p, C_v	heat capacity at constant pressure or volume, J/kg K
$\dot{m}$	mass flow, kg/s
N	revolution per minute, rpm
p	pressure, bar
Q	thermal energy, J/cycle
$\dot{Q}$	thermal power, W
R	gas constant, J/kg K
T	(absolute) temperature, K
t	time, s
V	volume, m ³
V_{swc}, V_{swe}	compression and expansion swept volumes
V_{clc}, V_{cle}	compression and expansion clearance volumes
z	gas compressibility factor ($=p/\rho RT$)
W	work, J/cycle
$\dot{W}$	mechanical power, W
W^*	work parameter ($=W/p_{max}V_T$)
ε	fractional temperature difference ($=\min(\varepsilon_K, \varepsilon_H)$)
η	cycle efficiency
η_C, η_E	compression and expansion polytropic efficiencies
η_{ideal}	ideal cycle efficiency ($=1 - T_K/T_H = 1 - \tau$)
η_R	thermal efficiency of the regenerator
ρ	density, kg/m ³

ϕ	empirical parameter for the calculation of the pressure drops (Equation (8))
φ	phase angle
ω	angular velocity ($=2\pi N/60$)

Subscripts

cr	critical point of the working fluid
max	maximum
min	minimum
r	reduced conditions ($T_r = T/T_{cr}$, $p_r = p/p_{cr}$)
C	compressor
E	expansion
R	regenerator
H	heater
K	cooler
T	total
D	dead
CK	denotes a physical quantity on the interface between the compression volume and the regenerator
KR	refer to a physical quantity on the interface between the cooler and the regenerator
RH	indicate a physical quantity on the interface between the regenerator and the heater
HE	relative to a physical quantity on the interface between the heater and the expansion volume

As a rule, an increase of the mean operating pressure of the engine raises the useful work and an increase of the speed of the engine raises the useful power, but, if the engine is ideal and the working fluid is an ideal gas, the power parameter $W^* = W/p_{max}V_T$ is constant.

Some Authors, [9], considered the possibility of using two-components and two-phases working fluids, aiming to a high specific output at moderate mean pressures.

On principle, an increase of the specific work is also obtainable by means of chemically reactive working fluids, as, for example, dissociating gases, like N_2O_4 , proposed in [10] for Brayton cycles.

The Malone engine, [11,12], is the well-known regenerative engine, similar in all to the Stirling hot-air, but employing water as heat transfer medium instead of air. In the Malone engine the very low compressibility of the liquid led to very high maximum pressures (about 800–1000 bar), both the swept volume of the pistons and the rotational speed (30–300 rpm) were small.

The potential advantages in using water as a working fluid are his good heat transfer coefficients and his high specific heat. The seal problems are likely easier to handle despite the very high pressures involved.

According to [11,13], the furnaces of the Malone engines manufactured between 1925 and 1927 were stoked with coal and coke. Therefore it is likely that the temperature of the working fluid in the heater exceeded the critical temperature of the water (373 °C). Malone, [11], tested many liquids and, among them, carbon dioxide (critical temperature of about 31 °C). So, in any case, it is likely that the working fluid was a dense gas rather than a liquid.

If an engine works with a dense gas instead of a liquid, at suitable volumes variation and round per minutes, the maximum pressures are lower than that in the case of a liquid, and, at the same time, the power parameter $W^* = W/p_{max}V_T$ would be high enough. On condition that a suitable working fluid is found.

Moreover, if the engine operates in regions of the thermodynamic plane where severe real gas effects prevail, it is possible to obtain good efficiencies even at moderate high temperatures (with assured

technological advantages), and where and when the working fluids behaving as ideal gases cannot assure acceptable performances.

2. The numerical model and the assumptions of calculations

There are many mechanical arrangements for the Stirling engines: the so called alpha-type, with two pistons; a piston and a displacer (in the same cylinder, the beta-type, or in two different cylinders, the gamma-type engine); some kinds of hybrid configurations, as the Ringbom engine, [15]. Yet, in any case, the basic design parameters of the engine are:

- the minimum operating temperature T_K , at the cooler, and the hot temperature T_H at the heater
- the swept compression volume V_{swc} and the swept expansion volume V_{swe}
- the total dead volume V_D
- the phase angle φ of the expansion space volume variations relative to the compression space volume variations
- the speed of the engine N

In the following, we will consider a two piston machine and a reference geometrical configuration with five different volumes, Fig. 1, [16]:

1. the compression volume V_C containing the working gas at the temperature T_C
2. the cooler volume V_K
3. the regenerator, with a volume V_R
4. the heater volume V_H
5. the expansion volume V_E containing the gas at temperature T_E

The cooler, the regenerator and the heater are interposed between the two opposed pistons, coupled to a common crankshaft, and the temperature of the gas, in the different operating volumes, is increasing from the minimum value (in the

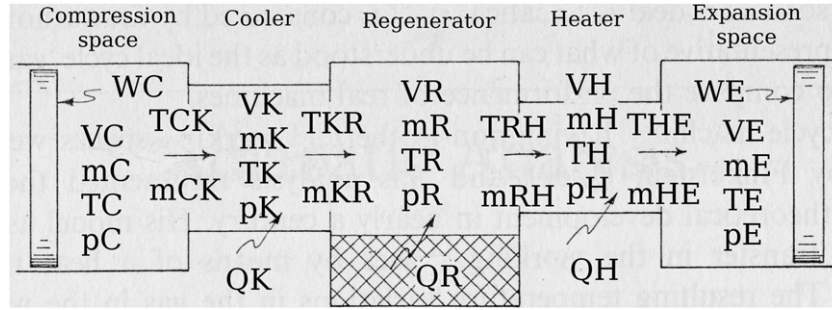


Fig. 1. Mechanical configuration with five components and with opposed pistons used for the calculations. Adapted from [16].

compression zone and in the cooler) to the maximum one (in the heater and in the expansion zone).

The compression volume V_C and the expansion volume V_E change in function of time t according to, [16]

$$\begin{aligned} V_C &= V_{clc} + \frac{1}{2}V_{swc}[1.0 + \cos(\omega t)] \\ V_E &= V_{cle} + \frac{1}{2}V_{swe}[1.0 + \cos(\omega t + \varphi)] \end{aligned} \quad (1)$$

in which, $\omega = 2\pi N/60$ is the angular velocity; V_{clc} and V_{cle} are respectively the clearance spaces in the compressor and in the expansion volumes; V_{swc} and V_{swe} are on the contrary the two swept volumes.

The total volume of the engine, V_T is then assumed equal to

$$V_T = V_{clc} + V_{swc} + V_K + V_R + V_H + V_{cle} + V_{swe} \quad (2)$$

and the dead volume V_D is considered as

$$V_D = V_{clc} + V_K + V_R + V_H + V_{cle} \quad (3)$$

The quantities to be evaluated at any instant of time, on a time interval equals to the period of the engine, are

- the pressure p in all the compartments of the engine
- the gas temperature T_C in the compression volume
- the gas temperature T_E in the expansion volume
- the gas mass flow $\dot{m}_{CK}$ from the compressor to the cooler (negative if the flow is from the cooler to the compressor)
- the gas mass flow $\dot{m}_{KR}$ from the cooler to the regenerator (negative if the flow is from the regenerator to the cooler)
- the gas mass flow $\dot{m}_{RH}$ from the regenerator to the heater (negative if the flow is from the heater to the regenerator)
- the gas mass flow $\dot{m}_{HE}$ from the heater to the expansion volume (assumed negative for the reversed flow)

Two further unknown parameters are the temperature T_{RK} of the gas flowing from the regenerator to the cooler and the temperature T_{RH} of the gas flowing from the regenerator to the heater over the cycle. If there are no pressure losses on the heat exchangers, the pressure p is the same in all the volumes; if the working gas is an ideal gas and the regenerator is ideal, then $T_{RK} = T_K$ and $T_{RH} = T_H$.

The useful work over the cycle is $W = W_E - W_C = \oint (\dot{W}_E - \dot{W}_C) dt$ and the cycle efficiency is evaluated as $\eta = W / \oint \dot{Q}_H dt = W / Q_H$.

The equations available are the usual equation of conservation of the mass and of the energy, applied to the whole machine and to every single volumes. The solution of the system of the differential equations gives the engine pressure p , the gas temperature T_C in the compression space, the gas temperature T_E in the expansion volume and all the mass flows. Fixed the pressure drops and T_{RK} , the solution of the system must be repeated at various T_{RH} ($\leq T_H$),

until the net thermal energy Q_R exchanged by the gas in the regenerator on a complete cycle is zero.

In the next subsections are illustrated the calculation assumptions.

2.1. The thermal efficiency of the regenerator

The regenerator is an essential component of the engine to obtain an high thermodynamic efficiency. Its effectiveness may be accounted by a thermal efficiency η_R defined as the ratio between the heat transferred to the working gas over the cycle and the heat transferred to the working gas over the cycle when the regenerator is ideal. In ideal conditions the working fluid during his transit through the regenerator exchanges the maximum allowable thermal energy. In this situation one of the two temperature end differences, $(T_{RK} - T_K)$ or $(T_H - T_{RH})$, is zero.

If the regenerator is not ideal, there exists a minimum temperature end difference greater than zero and we can assume as an efficiency indicator, instead of η_R , the design parameter

$$\varepsilon = \min(\varepsilon_K, \varepsilon_H) \quad (4)$$

where

$$\varepsilon_K = \frac{T_{RK} - T_K}{T_H - T_K} \quad \text{and} \quad \varepsilon_H = \frac{T_H - T_{RH}}{T_H - T_K} \quad (5)$$

as a consequence, in the equations of conservation of the energy and of the mass, the temperature of the gas inside the regenerator and inside the cooler space have to be assumed respectively equals to $T_R = 0.5 \times (T_{RK} + T_{RH})$ and to $T_K = 0.5 \times (T_{RK} + T_K)$. The temperature of the gas inside the heater is $T_H = 0.5 \times (T_{RH} + T_H)$.

2.2. Polytropic expansion and compression efficiencies

The losses caused by fluid viscosity are accounted by means of the expansion and compression efficiency η_E and η_C . In an ideal cycle $\eta_E = \eta_C = 1.0$; in real processes of compression or expansion, according to the variation of the volume dV/dt , the expansion useful work and the compression necessary work result respectively lower and greater than the ideal ones. So, $\dot{W}_E$ and $\dot{W}_C$ begin

$$\dot{W}_C = p \frac{dV_C}{dt} \frac{1}{\eta_C} \quad \text{if} \quad \frac{dV_C}{dt} < 0 \quad (6)$$

$$\dot{W}_C = \eta_C p \frac{dV_C}{dt} \quad \text{if} \quad \frac{dV_C}{dt} > 0$$

$$\dot{W}_E = \eta_E p \frac{dV_E}{dt} \quad \text{if} \quad \frac{dV_E}{dt} > 0 \quad (7)$$

$$\dot{W}_E = p \frac{dV_E}{dt} \frac{1}{\eta_E} \quad \text{if} \quad \frac{dV_E}{dt} < 0$$

2.3. The pressure drops

For sake of simplicity, all the pressure drops on the heat exchangers are concentrated between compressor and cooler so that $p_K = p_R = p_H = p_E$ and the mean pressure in the compression space $p_{C, mean}$ over the cycle results greater than the mean pressure in the rest of the engine.

The pressure drops are first of all a function of the mass flows, but, for simplicity of calculation, we assumed the concentrated pressure drop constant over the time according to

$$\begin{aligned} \text{if } \dot{m}_{CK} \geq 0.0 \quad \text{then} \quad p_E &= p_C(1 - \phi N^2) \\ \text{if } \dot{m}_{CK} < 0.0 \quad \text{then} \quad p_E &= p_C(1 + \phi N^2) \end{aligned} \quad (8)$$

in which ϕ is a suitable empirical constant and N the speed of the engine.

2.4. Further model assumptions

The following further calculation hypotheses are assumed:

- no leakage
- compression and expansion are adiabatic ($\dot{Q}_C = \dot{Q}_E = 0$) processes
- the working gas exchange thermal power with the surroundings only in the cooler and in the heater
- the external temperature walls of the cooler and of the heater are at the same temperature of the gas

The variations of the gas density as a function of temperature and pressure and all the necessary thermodynamic properties are evaluated by means of the Peng–Robinson equation of state, [17], Chapter 8, valid for pure fluids and fluids mixtures.

$$p = \frac{RT}{v-b} - \frac{a}{v(v+b) + b(v-b)} \quad (9)$$

where

$$b = \sum_{i=1}^n x_i b_i$$

$$a = \sum_{i=1}^n \sum_{j=1}^n x_i x_j \sqrt{a_i a_j} (1 - k_{ij})$$

$$b_i = 0.07780 \frac{RT_{c,i}}{p_{c,i}}$$

$$a_i = \alpha_i(T) 0.45724 \frac{R^2 T_{c,i}^2}{p_{c,i}}$$

$$\alpha_i(T) = [1 + m_i(1 - \sqrt{T_{r,i}})]^2$$

$$m_i = 0.37464 + 1.54226\omega_i - 0.26992\omega_i^2$$

in which x_i are the molar fractions, $k_{i,j}$ are suitable constants, and ω_i is the acentric factor of the component i of the mixture. For pure fluids, $n = 1$ and $x_i = x_j = 1.0$ and $k_{ij} = 0.0$. The parameter R is the gas constant.

Then, from the volumetric behaviour, one can evaluate all the necessary thermodynamic properties of the working gas.

3. Results of the thermodynamic analysis of the ideal engine

The numerical procedure previously outlined was applied to helium (He), hydrogen (H₂), nitrogen (N₂) and argon (Ar), representative ideal gases at the ordinary temperatures, and to carbon dioxide (CO₂), trifluoromethane (HFC – 23, CHF₃), ethane (CH₄), sulfur hexafluoride (SF₆), pentafluoroethane (HFC – 125, CF₃ – CHF₂), difluoromethane (CHF₂). In Table 1 are some properties of the considered fluids. The parameters assumed for the calculations

Table 1

Molecular mass, critical temperature and critical pressure of the considered working fluids.

Fluid	Molecular mass (g/mole)	Critical temperature (K)	Critical pressure (bar)
Helium	4.003	5.201	2.275
Hydrogen	2.016	33.25	12.97
Nitrogen	28.01	126.2	33.98
Argon	39.95	150.86	48.98
Trifluoromethane HFC-23	70.01	298.97	48.36
Carbon Dioxide	44.01	304.21	73.83
Ethane	30.07	305.32	48.72
Sulfur Hexafluoride	146.06	318.72	37.60
Pentafluoroethane HFC-125	120.02	339.17	36.15
Difluoromethane	52.02	351.26	58.05

are in Table 2 (a) and (b). An ideal cycle efficiency rather low ($\eta_{ideal} = 1 - \tau = 0.5$) was assumed, to limit the maximum temperature T_H , on account of the thermal stability of the organic working fluids.

The geometrical and kinematical parameters in Table 2 are just for reference and we have assumed those of Viebach CHP unit presented in [18], page. 213. The basic thermodynamic distinctive characteristics of the cycles depend in fact only on the thermal and volumetric comportment of the working gas. Even though, τ , the geometric and the kinematic parameters control directly the value of W^* .

The critical temperature of the organic and inorganic considered fluids are such that they show severe real gas effects close to the standard environmental conditions. The critical pressure of CO₂ may be perhaps too high for the particular application here studied. Anyway, it was considered as a typical case of study.

Besides, carbon dioxide power cycles were extensively investigated in the past for applications where the final cooling of the working medium in a condenser takes place just below the critical temperature of the substance proposed, [22,23], and in recent years there was a revival in CO₂ cycles studies mainly connected with their perspective use in medium temperature advanced nuclear reactors, [24].

The refrigerant HFC – 23 was experimentally found stable in static tests at about 450 °C, [19] but its critical temperature of about 26 °C implies a heat rejection process at relatively low temperatures, which could not be easy to perform in a hot environment. A second stable refrigerant, the refrigerant HFC – 125, with a critical temperature of about 66 °C is suitable for cogeneration systems.

Table 2

Assumed parameters for the calculations of the performances of the thermodynamic cycles.

(a)	
$V_{swc} = V_{swe}$	0.5 dm ³
$V_{clc} = V_{cle}$	0.123 dm ³
V_K	0.0944 dm ³
V_R	0.4054 dm ³
V_H	0.2934 dm ³
V_D	1.0392 dm ³
V_T	2.0392 dm ³
$\tau = T_K/T_H$	0.5
N	800 rpm
ϕ	$\pi/2$
(b)	
IDEAL CYCLE	
$\eta_C = \eta_E$	1.0
$\varepsilon = \min(\varepsilon_K, \varepsilon_H)$	0.0
ϕN^2	0.0
REAL CYCLE	
$\eta_C = \eta_E$	0.95
$\varepsilon = \min(\varepsilon_K, \varepsilon_H)$	0.05
ϕN^2	0.00055

The sulfur hexafluoride is a fluid widely used in industrial processes due to its high thermal stability and low toxicity, [20]. But it is a potent greenhouse gas and, associated with stainless steel, its thermal stability at high temperature is questionable, [21]. It is considered here because its critical temperature (about 46 °C) is intermediate between that of the ethane and that of HFC – 125.

In any case, an effective way of obtaining a working fluid with an optimized critical temperature relying on the limited number of available substances is the use of mixtures, [14].

The Fig. 2.a represents the power parameter $W^* = W/p_{\max}V_T$ as a function of the maximum reduced pressure $p_{\max, r} = p_{\max}/p_{cr}$ for the working fluids listed in the Table 1. The cycles with ideal gas have $W^* \approx 0.035$ at every $p_{\max, r}$; the cycles working with gases displaying intense real gas effects at temperatures T_K have power parameters increasing from about 0.04 (in ideal gas conditions, at low $p_{\max, r}$) to about 0.052 at $p_{\max, r} \approx 1 \div 1.5$. At very high reduced maximum pressures (about 5.5 ÷ 6), the power parameter begins 0.1 ÷ 0.11: an increase, compared with that of the ideal gases, of 200%.

Calculated efficiencies for CO₂ ideal cycles are reported in the Fig. 2.b as a function of the maximum reduced pressure, for various reduced cooler temperatures $T_{K, r}$. As $T_{K, r}$ increases the real gas effects become less intense and the cycle efficiency tends to the value of that of the ideal cycle using an ideal gas. For example, at $p_{\max, r} = 2.0$, the cycle efficiency is 0.26 for $T_{K, r} = 1.1$ (power parameter $W^* = 0.058$), and 0.44 for $T_{K, r} = 2.0$ (power parameter $W^* = 0.04$). Just for comparison, in the Fig. 2.b is reported the efficiency of hydrogen cycles at $T_K = 303.15$ K. The efficiency is about 0.45 at every operating reduced pressure $p_{\max, r}$ and $W^* = 0.035$ (see the Fig. 2.a).

The cycle efficiency in Fig. 2.b initially decreases with the maximum reduced pressure because real gas effects introduce a thermodynamic loss in the regenerator: although the temperature difference $T_{RK} - T_K$ is zero ($\varepsilon_K = 0.0$) the variations in the specific heat of the working fluid are such that $T_H - T_{RH}$ results positive ($\varepsilon_H > 0.0$).

For example, with reference of the curve at $T_{K, r} = T_K/T_{cr} = 1.1$ of the Fig. 2.b, the cycle efficiency is 0.26 at $p_{\max, r} = 2$, where the fractional temperature difference $\varepsilon_H = (T_H - T_{RH})/(T_H - T_K)$ reaches its maximum value (see Fig. 2.c), and the cycle efficiency increases with $p_{\max, r}$ because ε_H decreases.

As a general rule, when the reduced cooler temperature increases the beneficial real gas effects on the net work become less important and the power parameter W^* decreases. On the contrary, the thermodynamic efficiency improves.

At very high reduced pressures, and at $T_{K, r}$ not too different from 1.0, the real gas effects are such that both the efficiency and the specific work are reasonably high. The maximum advantages are at $T_{K, r}$ near 1.0 and at high $p_{\max, r}$. In those conditions the power parameter and the thermodynamic efficiency actually may be very high.

As an example, the cycle (1) in Fig. 2.d, at $p_{\max, r} = 6.05$ and $T_{K, r} = 1.01$, has an efficiency of 0.46 with $\varepsilon = \varepsilon_C = \varepsilon_H = 0.0$; the cycle (2), at $p_{\max, r} = 1.06$, has $\varepsilon_C = 0.0$, but ε_H results 0.20 ($(T_H - T_{RH}) = 61$ °C) and, as a consequence, a lower thermodynamic efficiency of 0.23. In both cases, $T_H/T_K = 0.5$. The power parameter is 0.1 for the cycle (1) and 0.05 for the cycle (2). In the Fig. 2.d is reported, just as a comparison, an ideal hydrogen cycle with $T_K = 303.15$ K. His efficiency is good, similar to that of the CO₂ cycle (1), but his power parameter W^* results equal only to about 0.035.

The remarkable difference in W^* between real gas and ideal gas cycles is due to the high reciprocal isothermal compressibility $(dp/dQ)_T$ of dense gases at high pressures while the good efficiency of real gas cycles at very high pressure is a result of the distinctive variability of the heat capacity of the working fluid.

In an ideal gas the heat capacities at constant volume C_V and at constant pressure C_p are only a function of the temperature while for a real gas they are also a strong function of the pressure. As a consequence, if the regenerator is ideal (as considered in our analysis) and the working fluid is an ideal gas, then $T_{RK} \rightarrow T_K$ and

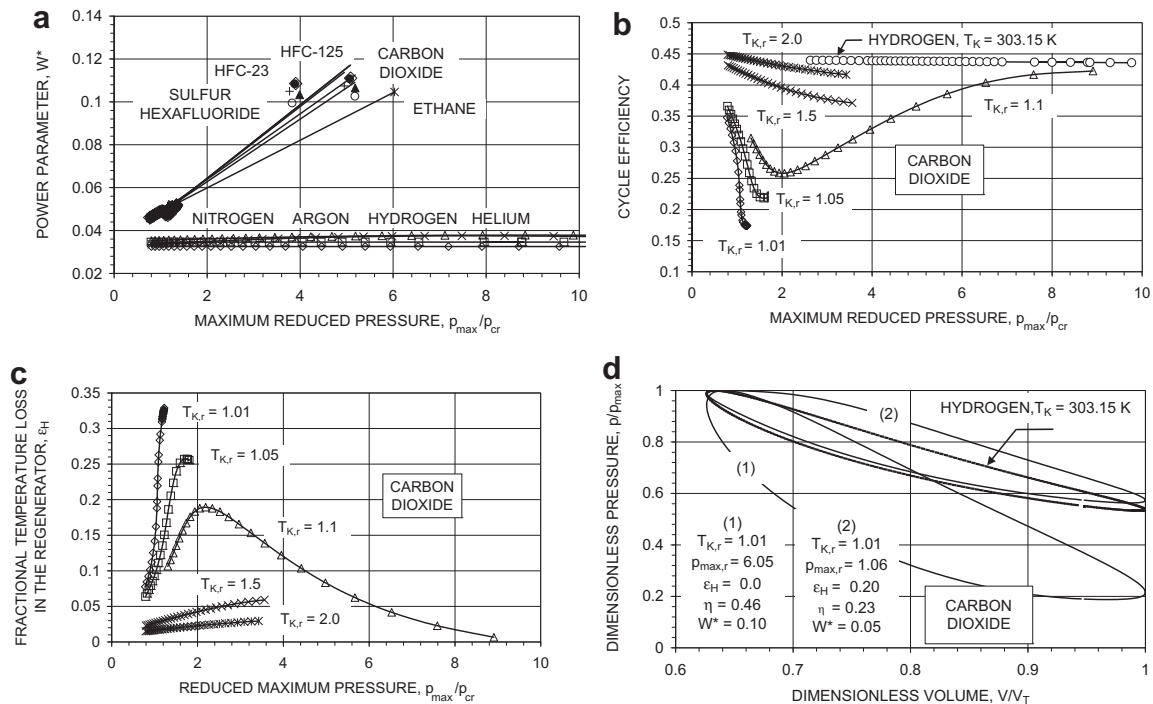


Fig. 2. Results for ideal cycles: $\eta_C = \eta_E = 1.0$, $\varepsilon = \min(\varepsilon_K, \varepsilon_H) = 0.0$, $\phi = 0.0$. The cycles with carbon dioxide, ethane, HFC – 125, HFC – 23 and sulfur hexafluoride are evaluated at $T_{K, r} = 1.01$; the cycles using nitrogen, hydrogen, argon and helium are calculated at $T_K = 303.15$ K. (a) Power parameters W^* as a function of the maximum reduced pressure for some working fluids. (b) Thermodynamic cycle efficiency as a function of the maximum reduced pressure at various $T_{K, r}$ for carbon dioxide and hydrogen. (c) Fractional temperature loss ε_H in the regenerator as a function of the maximum reduced pressure at various $T_{K, r}$ for carbon dioxide. (d) Pressure–volume diagrams for carbon dioxide cycles at $T_{K, r} = 1.01$ and at two maximum reduced pressures and for hydrogen at $p_{\max} = 100$ bar.

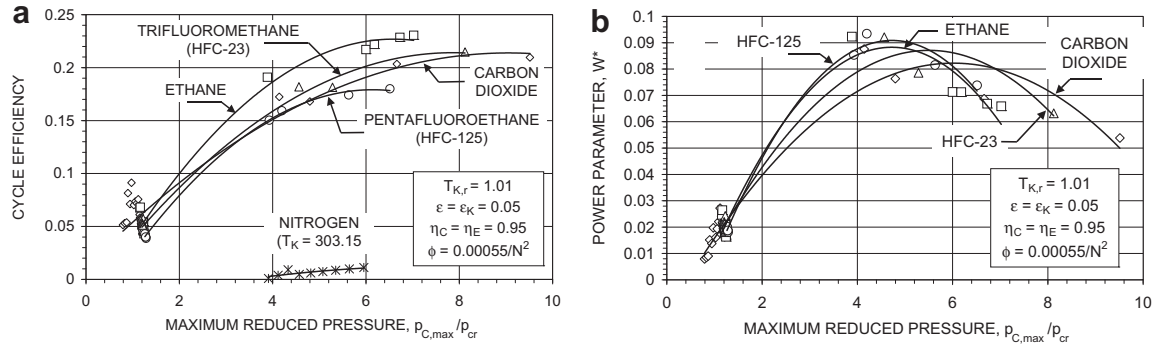


Fig. 3. Cycle efficiency (Fig. 3 a) and power parameter (Fig. 3 b) for real cycles operating with various working fluids as a function of the maximum reduced pressure. The nitrogen cycle was calculated at $T_K = 303.15$ K. At this operating temperatures nitrogen behaves as an ideal gas. Cycles operating with the other fluids were calculated at $T_K = 1.01 \times T_{cr}$. The assumed polytropic efficiencies were $\eta_C = \eta_E = 0.95$. Pressure drops were accounted by the parameter $\phi = 0.00055/N^2$.

$T_{RH} \rightarrow T_H$. If the regenerator is ideal but the working gas shows real gas effects, there are two possibilities: (a) the temperature $T_{RK} = T_K$ and, as a consequence, $T_{RH} < T_H$; (b) the temperature $T_{RK} > T_K$ and, as a consequence, $T_{RH} = T_H$.

In other words, even if the regenerator is ideal, the compressibility effects on the specific heats produce a $\Delta T_{KR} = T_{RK} - T_K \geq 0$ or a $\Delta T_{RH} = T_H - T_{RH} \geq 0$ in the regenerator: one of the two ΔT at the end of the regenerator is invariably greater than zero.

However, at very high pressure the real gas effects are such that the heat capacities result chiefly, once again, a function only of the temperature, and as a consequence $\Delta T_{RH} \rightarrow \Delta T_{KR}$ or, if the regenerator is ideal, $\Delta T_{RH} = \Delta T_{KR} = 0.0$.

4. Results of the thermodynamic analysis of the real engine

The parameters η_C , η_E , ϕ and ε used to calculate the thermodynamic performances of real engines are reported in Table 2(b).

Owing to the assumed simplifying pressure drops model, the pressure in the compressor space p_C is greater than that in the cooler, regenerator, heater and in the expansion spaces. That is: $p_C \geq p_K = p_R = p_H = p_E$.

In the Fig. 3 some calculation results of cycle efficiencies are reported for nitrogen (in Fig. 3.a), carbon dioxide, ethane, HFC – 23 and HFC – 125. For the cycles operating with nitrogen, calculated at $T_K = 303.15$ K, the component inefficiencies are such as the cycle efficiency results practically zero at any maximum pressure (for example, 1.52% and 1.11% at $p_{C,max,r} = 6$ and, respectively, $\varepsilon = \varepsilon_K = 0.0$ and 0.05). With the assumed parameters and in the considered range of reduced maximum pressure, the efficiencies of real cycles operating with helium and hydrogen result also practically equal to zero and are not reported in the Figure. Cycles operating with the other working fluids have been calculated at $T_{K,r} = 1.01$.

In any case, at low maximum pressure $p_{C,max}$ the cycle efficiencies are very low (from 5 to 10%, depending on the fluid);

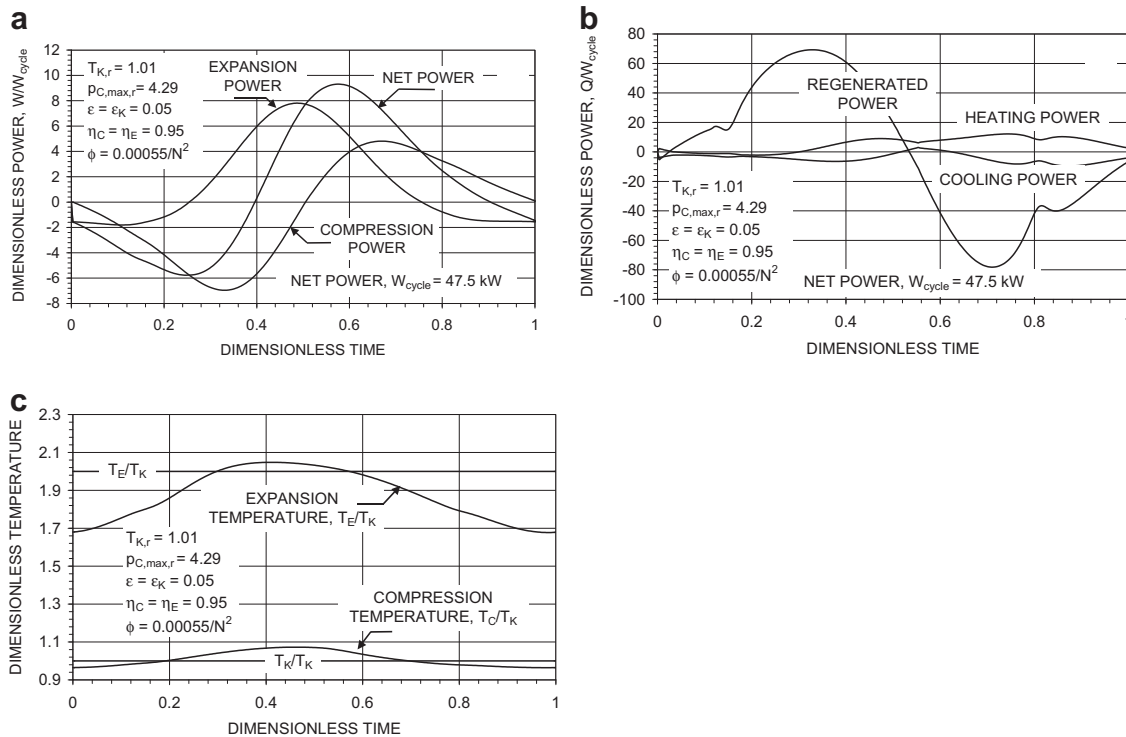


Fig. 4. Some calculation results for a cycle operating with ethane. The cooler temperature $T_K = 308.37$ K ($= 1.01 \times T_{cr}$), the heater temperature T_H is $T_K/\tau = 616.75$ K. (a) Time variations of the transferred mechanical powers. (b) Variations with time of the transferred thermal powers. (c) Dimensionless expansion and compression gas temperature as a function of the dimensionless time.

increasing the working pressure the cycle efficiency increases because the component inefficiencies become less important due to the very great positive compressibility effects on the thermodynamic of the cycle.

The thermal loss in the regenerator, by itself, imposes a decrease in cycle efficiency of about 10–20 points, according to the fluid and the maximum operating pressure. A fractional temperature difference $\varepsilon = \varepsilon_K$ of 0.05 corresponds to a minimum temperature difference $(T_{R,K} - T_K)$ of about 15–17 °C, depending on the critical temperature of the considered working fluid.

The Fig. 3.b represents the power parameter W^* as a function of the maximum reduced pressure for four fluids working at $T_K, r = 1.01$. The optimum W^* is 0.08–0.09 at about $p_{C, max, r} = p_{C, max}/p_{Cr} = 5$ for all fluids.

The Fig. 4 refers to ethane, at $p_{C, max, r} = 4.29$. The calculated cycle efficiency results 19% and the power parameter W^* is 0.084. The mean pressures in the compressor space and in the expansion space are respectively 120 bar and 100 bar. The maximum ones are 209 and 190 bar.

The mechanical powers, normalized by the net power cycle, are reported in Fig. 4.a. The net power over the period changes between +9.3 and –5.8 times the net power cycle. The normalized thermal powers are in Fig. 4.b. The maximum regenerated power is about 70 times the useful power, a further demonstration of the essential role of the regenerator also with real gas working fluids. Finally, in Fig. 4.c are reported the variations of the gas temperatures in the (adiabatic) compression and expansion volumes. The expansion temperature changes over the period of 22%; the compression temperature changes of only about 11%.

5. Conclusions

The results of the analysis carried out in the previous sections show that, above all, the real gas effects (in special mode the low gas compressibility at high reduced pressures) are responsible for very high power parameters W^* in comparison with the values obtainable from cycles operating with ideal gases: 2–3 times, at T_K near environment conditions, for example, for ideal cycle, at $p_{max, r}$ of about 5 and $\tau = T_K/T_H = 0.5$.

On the other hand, for ideal cycles ($\eta_C = \eta_E = 1.0$, $\varepsilon = \phi = 0.0$), the real gas effects introduce a thermal irreversibility in the regenerator that lowers the thermodynamic efficiency. However, this negative effect decreases with increasing the maximum operating pressure. For example, the thermodynamic efficiency of the considered ideal cycles with carbon dioxide, at $T_K, r = 1.1$ is 0.25 at $p_{max, r} = p_{max}/p_{Cr} = 2$, and rises to 0.4 at $p_{max, r} = 6$ (ideal cycle efficiency $\eta_{ideal} = 1 - \tau = 0.5$).

So, at very high operating reduced pressures and with $T_K, r \approx 1$ the real gas effects allow, in ideal cycles, high efficiency and at the same time high power parameters.

The situation changes impressively introducing the component efficiencies: the real gas effects, although partly theoretically detrimental in ideal cycles, give important efficiency gains in real cycles. For example, efficiencies of about 20%, with power parameters of 0.07–0.09 are obtainable for cycles at $T_K, r = 1.01$, $\tau = 0.5$ and reasonable component efficiencies. At the same operating conditions (high τ , relatively low regenerator effectiveness and relatively high pressure drops) engines with ideal gases, like hydrogen, helium or nitrogen, do not attain useful cycle performances.

In order to make the critical region accessible to the compression process, the working fluid critical temperature must have a well defined value: only organic fluids offer a wide selection of critical temperatures for different technical applications, but owing to the limited thermal stability of organic substances, only cycles at top temperature in the range of 300–400 °C are feasible and this

has a positive influence on material problems too: ordinary carbon steel could be an adequate choice even for the hottest engine part. However, the thermal stability could be negatively affected by contaminants, like lubrication oil.

On the other hand, the good energy performance of real gas cycles is obtained at the cost of high maximum cycle pressure, in the range of at least 100–300 bar. These high pressures nevertheless have large positive effects on the heat power transferred per unit of pumping mechanical power. In fact, in regard to the heat transfer properties and to the power consumed by friction, we may observe that, as reported for example in [25], page. 445, the heat transfer coefficient h (the transferred heat power per unit of surface and for one degree of temperature difference) and the friction power $\dot{W}_f$ per unit of surface area may be represented by Equations (10) and (11).

$$h = \frac{\mu C_p}{N_{Pr}^{2/3}} \frac{1}{4r_h} \Gamma_h(N_{Re}) \quad (10)$$

$$\dot{W}_f = \frac{1}{2} \frac{\mu^3}{\rho^2} \left(\frac{1}{4r_h} \right)^3 \Gamma_f(N_{Re}) \quad (11)$$

in which r_h represents the hydraulic radius, N_{Re} is the Reynolds number, $N_{Pr} (= \mu C_p/k)$ is the Prandtl number, μ the gas viscosity and k the thermal conductivity. The functions $\Gamma_h(N_{Re})$ and $\Gamma_f(N_{Re})$ are convenient functions depending of the geometry of the heat exchanger.

Then, fixed the mean operating reduced pressures and temperatures, the geometry and the Reynolds numbers, the ratio of $(h/\dot{W}_f)^*$, for a gas which behaves as a real gas (compressibility factor $z = z^*$ less or greater than 1.0), and $(h/\dot{W}_f)^\circ$ for the same gas in ideal conditions ($z = z^\circ = 1.0$) results

$$\frac{(h/\dot{W}_f)^*}{(h/\dot{W}_f)^\circ} = \left(\frac{z^\circ}{z^*} \right)^2 \left(\frac{\mu^\circ}{\mu^*} \right)^3 \left(\frac{k^*}{k^\circ} \right) \left(\frac{N_{Pr}^*}{N_{Pr}^\circ} \right)^{1/3} \quad (12)$$

for a fluid at mean reduced temperatures of about 1.5 and mean operating reduced pressures of about 5, the ratio of Equation 12 may easily, roughly, be of the order of 10.

The design and manufacture of a machine operating at very high pressure (although at moderate maximum temperature) can pose technological problems, but the high W^* due to the low compressibility of the gas allows the realization of small engines with significant useful power and this (a) mitigates the structural problems, (b) permits the pressurization of the crankcase, with advantages in the seals realization.

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